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THE RESOLUTION OF FREQUENCY MEASUREMENTS IN PFM TELEMETRY

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

This article outlines a method for high resolution measurements of pulse frequency modulation telemetry signals mixed with additive Gaussian noise. The relations between the resolution and the signal-to-noise ratio and between the resolution and the satellite range are given.

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INTRODUCTION

Present techniques in the data processing area of Goddard Space Flight Center do not provide highly resolved measurements of pulse frequency modulation (PFM) telemetry data, because such measurements were not necessary until recently. New requirements have led to the development of equipment which will provide highly resolved PFM telemetry data measurements in accord with the signal-to-noise ratio (S/N) of that data. The ensuing discussion outlines the methods employed by this new equipment, its performance, and some of the considerations affecting its conception.

PULSE FREQUENCY MODULATION TELEMETRY

Pulse frequency modulation is one of the standard telemetering systems used on Goddard Space Flight Center (GSFC) satellites (Reference 1). Conventional PFM is a system whereby the information of each channel in a telemeter determines the frequency of a sine-wave burst whose duration is nominally 10 msec. (The telemeter encoder output is a square wave, but by the time the signal gets to the data reduction center it more closely resembles a sine wave.) Each 10 msec burst is preceded by a 10 msec blank. Synchronism of the system is maintained by a synchronizing burst which is 15 msec long preceded by a 5 msec blank. The frequency of these bursts normally can be anywhere in the range of 5 to 15 kc.
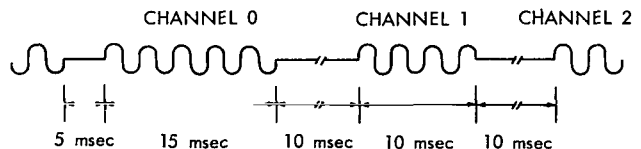

Figure 1—The PFM Signal.

PRESENT PFM DATA PROCESSING TECHNIQUES

It has been standard practice at GSFC to process PFM by the use of contiguous filter banks, often called comb filters. Each of these narrow band filters is separated by 100 cps in center frequency from its neighbors and has a nominal 3 db bandwidth of 100 cps. The frequency of the

signal within the PFM burst is said to equal the center frequency of the filter whose output envelope detector is maximum at the end of the burst time.

The advantage of such a filter bank system in PFM data processing is the S/N improvement. This improvement results for this reason: the bandwidth of the noise mixed with the signal going into the filter bank is 10 kc. Not considering the sidebands, the data signal will occupy a single frequency anywhere in the 10 kc band. The technique employed is that of recognizing in which filter the data signal resides and then to gate off all other filter detectors so that in effect the bandwidth of the noise accepted by the comb filter output is constrained to the noise bandwidth of the individual responding filter.

One of the major disadvantages of this system is that we only know the incoming frequency to be 1 of 100 possibilities, no matter how high the input S/N, so long as it is above the system threshold of operability. Thus it is said that the incoming frequencies are quantized to 1 percent. Figure 2 illustrates this statement. All errors from plus to minus 50 cps ($\pm 1/2$ percent of 10 kc) are assumed equally likely and the standard deviation of the error is 28.8 cps.
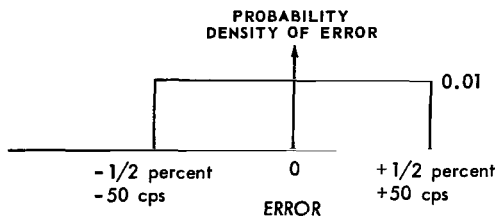
Figure 2—Probability density of error for a system quantized in 100 cycle increments throughout a 10 kc band.

With such a rectangular distribution, it is possible simply by inspection to see the percent of the time that the errors lie within any specified range; for example, 50 percent of the time the error is less than ± 25 cps ($\pm 1/4$ percent). With this filter bank principle, smaller quantization can be obtained only by using more filters with narrower bandwidths. This becomes not only excessively expensive, but also operationally cumbersome. For example, a 0.1 percent quantum size would require 1000 filters each with a bandwidth of 10 cps. This would introduce the complication that the build-up time for a 10 cps filter would be greater than 10 msec, and therefore not compatible with standard PFM.

When more accurate measurement of the incoming frequencies is necessary, a means other than the filter bank approach should be employed. The measurement method should take advantage of the quality of the signal that is available; i.e., when the S/N is high, very accurate measurements should be possible.

In the ensuing discussion the word "resolution" is used in the following way. The frequency f of an incoming sine wave is to be measured and it is known to lie within two bounds, f_{max} and f_{min}. The measurement, which really gives us only an estimate of the incoming frequency, is disturbed by additive Gaussian noise. This measured value (or estimate) is called $\hat{f}$. The quantity $f - \hat{f}$ is a Gaussian-distributed random variable with zero mean (under the assumption that the instrumentation does not introduce a statistical bias). The value of this quantity at K standard deviations is called c. By definition, c is resolution. A specification of resolution must be accompanied by a specification of K. The c value is in cps, but resolution also can be thought of in terms of percent. The total interval between f_{max} and f_{min} is B cps. If B is divided into $100/m$ equal parts each one c cps in size, m is by definition the percent resolution (i.e., $m = 100\ c/B$).

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Some examples which may explain this use of the term "resolution" better are given in Appendix A. Figure 3 shows the relation between K* and the probability that a value lies within $\pm K\sigma$ of a Gaussian distribution. Contrary to common sense the term *high resolution* in this context, means that the *resolution* c or m *is small*.

A HIGH RESOLUTION PROCESSOR

Commercial counters are not adequate for highly resolved measurements of frequency when the time available for measurement is short and the input is perturbed by noise. Equipment that measures events per fixed unit of time can make the measurement to only ± 1 event per sample; for a PFM 5 kc signal, having a 10 msec sample time this is ± 1 cycle per 50 cycles, which amounts to ± 2 percent quantum size. The quantum size (percent), hence the resolution, is frequency dependent. Equipments that measure elapsed time per fixed number of events also have the limitation that the resolution of the measurement is dependent upon the input frequency (Reference 2).

To circumvent the inherent limitation of both standard methods of frequency measurements, a device called a high resolution processor (HRP) has been designed and implemented.

General Principles

The HRP measures the elapsed time (to within 0.2 μsec) of y cycles, where y is the integral number of cycles plus 1 that elapse in a preset time interval (Reference 3). The output of the device is 2 binary (or BCD) numbers, one indicating y, and the other indicating z, the number of 5 Mc pulses counted in exactly y cycles. The two numbers are then operated on to yield the frequency,

$$\hat{f} = \frac{y}{z} (5 \times 10^6) \text{ cps }.$$

This operation can be done in either a small internal divider or a general purpose computer; the output is on digital magnetic tape.

The HRP measures frequency, over nearly the same preset length of time, for a range of frequencies which is in principle unlimited, unlike the standard methods mentioned which measure
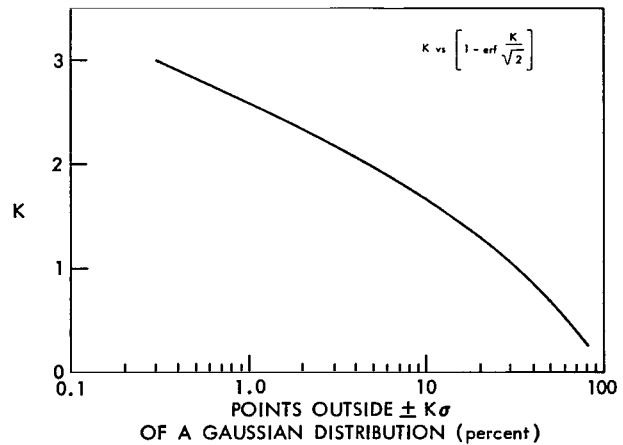

Figure 3—The error function (K vs. 1 – erf K/$\sqrt{2}$).

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*K/$\sqrt{2}$ is the argument of the error function, abbreviated erf (K/$\sqrt{2}$).

over a constant number of cycles or a constant interval (without regard to having an integral number of data cycles) for any frequency. The HRP is designed to process data stored on magnetic tape. In order to compensate for tape speed variation the processor clock frequency, which provides the count z, is generated by a frequency multiplier whose input is a reference frequency from the magnetic tape. Figure 4 is a simplified functional diagram of the high resolution processor.

Quantization Error in the HRP

The analysis of the quantization error (i.e., the error inherent to the measurement technique itself, which exists whether or not noise is on the input signal) is given in Appendix B. This error is usually small compared with the error caused by noise perturbing the input signal. The lower limit on the resolution m of this system is due to this quantization error. This lower limit is dependent upon frequency, and for the two extremes of frequency, 5 and 15 kc, it is 0.00095 and 0.00285 percent respectively. At other frequencies within the band B the resolution will lie between these bounds.

Figure 4—Simplified functional diagram of the high resolution processor.

Resolution of the HRP

Reference 2

"relates the resolution to which the frequency of a sine wave can be measured (using zero-crossing* techniques) to the signal-to-noise ratio of the wave and the time available to make the wave measurement. It demonstrates the need for making measurements over a constant time interval ... if the resolution of measurement is to be independent of frequency."

A device, such as the HRP, which makes frequency measurements over a nearly constant time interval regardless of frequency, adheres to the relations developed in Reference 2:

$$\frac{S}{N} = \frac{K}{2\pi T}\left(\frac{f + c}{cf}\right) \tag{1}$$

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*The expression "zero-crossing" is used here to mean the time of the occurrence of the excursion of the signal voltage through the dc level of the signal.
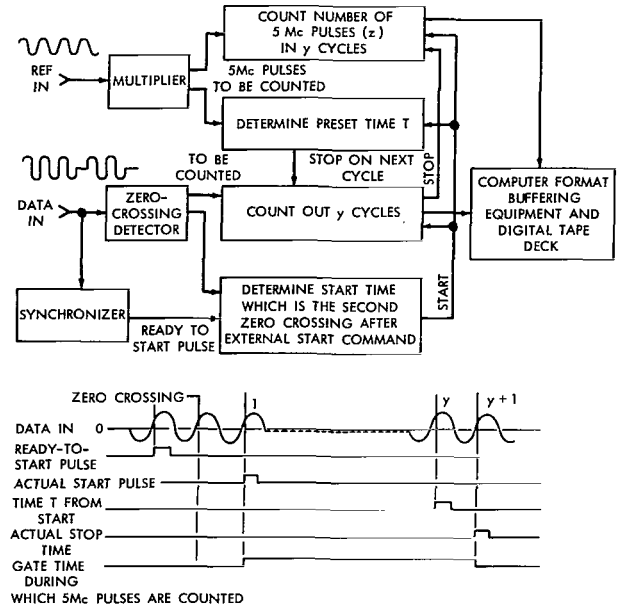
or

$$c = \frac{1}{\frac{S}{N}\frac{2\pi T}{K} - \frac{1}{f}} ,$$

(2)

where S/N is the signal-to-noise voltage ratio at the input and T is the length of time over which the measurement is made.

These relations were developed from a mathematical model which assumes that the error in determining the frequency is due to the uncertainty with which the time of zero crossing can be detected. This uncertainty is caused by additive Gaussian noise. It has been assumed that other sources of error (such as circuit noise) are negligible, and that only n + 1 positive-going zero crossings accompany n cycles of the actual signal.

Equations 1 and 2 have been demonstrated by tests run on the HRP whereby S/N , T, and f were varied in turn and the data gathered was statistically analyzed in a general purpose computer. The output of the computer facilitated a plot of the percentage of data points outside of the specified resolution vs. S/N , in families of T and f.

Figure 5 illustrates the test results for three sets of readings, one taken at a data frequency of 5 kc, one at 10 kc, and one at 15 kc. The plotted test results are based on the percentage of frequency readings which fell within ±10 cps ($c = 10$) of the true data frequency for various S/N. The continuous curve is a calculated curve for $c = 10$ cps.

Each plot point represents a test of 4000 data points where, for the entire test, the input signal of precise frequency and constant amplitude was linearly summed with band-limited, white Gaussian noise whose rms voltage remained constant. The 4000 data points (readings y and z) constituted one file of data in computer format on magnetic tape. Each file was processed by the computer to develop the percentage of data points which fall outside the limits of ±10 cps from the true data frequency.

Inspection of Figure 5 shows that all but two plot points fall within ±0.5 db of the calculated curve. The two points which exceed this deviation occurred at a data recovery rate which generally would not be of interest. They fell beyond the S/N ratio where 30 percent of the data readings would be reliable. The technique used in implementing the zero crossing of the signal becomes unreliable as the S/N becomes small, because excessive noise will cause a high incidence of erroneous zero crossings by producing more than 1 positive crossing per signal cycle. Thus, the resultant count of cycles in the reading time would be too high.

The test results of Figure 6 support the relation of Equation 1 whereby a lower S/N is required for a fixed time interval T when the resolution of measurement is decreased (i.e., c is larger). The calculated curves for each of two resolutions are shown. Again the plot points of experimental data represent 4000 data point samples. The frequency f and the time interval T for these tests were held constant at 10 kc and 5.0 msec respectively.
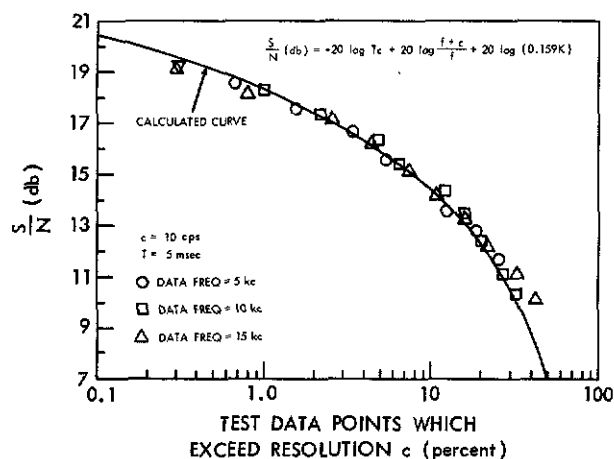
Figure 5—Percentage of test data points which deviate from the true frequency by more than +10 cps as a function of S/N.
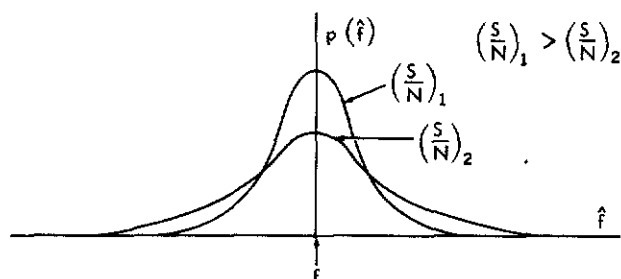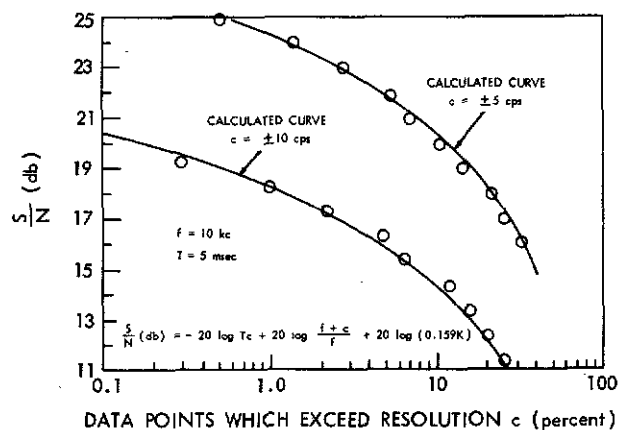

Figure 6—Percentage of test data points which deviate from the true frequency by more than c cps as a function of S/N.

Figure 7—Probability density of frequency readings for two S/N ratios.

Another test was made, similar to the test described above except that T was extended and the resolution c was held constant. The results confirmed the relation of Equation 1. By extending the time interval from the 5.15 msec used in the first portion of the test to 10.15 msec, the expected -5.9 db* change in S/N was found in all cases to be less than 0.5 db in error.

The probability density distribution of the experimental data points was Gaussian in nature, because the noise perturbing the readings was Gaussian. A sketch of these distributions is shown in Figure 7 illustrating that as the S/N is decreased the distribution curve widens.

Improving the HRP Performance by Preprocessing

If the resolution obtainable from the HRP for a specific S/N is inadequate, it can be improved by inserting a system ahead of the HRP which acts on the input signal to improve the S/N. This principle is presently being studied. Both filter banks and phase-locked loops have limitations in this application. They both need time to "acquire" the incoming frequency, time that would be valuable in improving the resolution, as illustrated in Equation 2. The trade-off between the improvement in S/N and measurement time is now being considered.

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*$20 \log \left(T_1/T_2 \right) = 20 \log (5.15/10.15) = -5.9$ db.

Determining the S/N at the HRP Input from Resulting Data

If it is known that for some interval of time the signal maintains a constant frequency but is perturbed by an unknown amount of Gaussian noise, and if an adequately large sample can be taken by the HRP, then by statistical analysis the true signal frequency and the S/N ratio at the input can be determined within known tolerances.

Figure 8 gives a plot of S/N versus c for three coefficients of confidence. The K = 1 line serves the special purpose of enabling the determination of the S/N from analysis of the resultant data. From an adequately large sample of constant frequency data the standard deviation of the readings can be calculated. This is the value of c for K = 1. The intersection of this value with the K = 1 line indicates the S/N of the HRP input during processing.
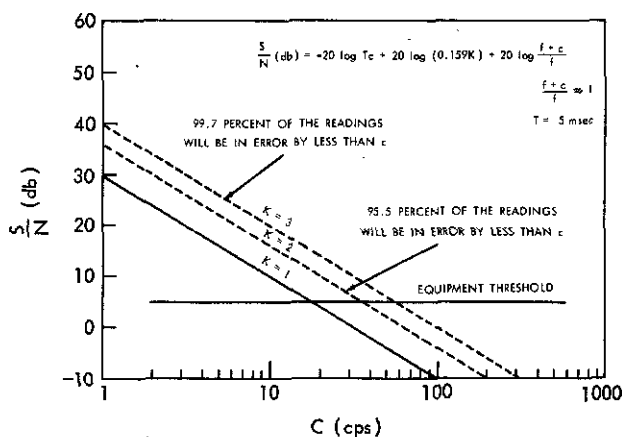

Figure 8—S/N vs. c for three values of K.

When the S/N of the HRP input is known the value of c for 95.5 percent confidence and 99.7 percent confidence can be read from the K = 2 and K = 3 curves, respectively.

THE RELATION BETWEEN RESOLUTION AND THE PARAMETERS AND RANGE OF A SATELLITE

The percent resolution (m) of the data, as a function of the distance between the satellite and the receiving station, can be plotted for a desired confidence level of data retrieval if the following are known: (1) the power of the satellite transmitter, (2) the gain of the transmitting antenna, (3) the gain of the receiving antenna, (4) the frequency of the carrier, and (5) the noise power at the receiver output. Equation 2 may be expressed

$$m = \frac{100}{\frac{2\pi TB}{K}\left(\frac{S}{N}\right)_{rms\ voltage}} - \frac{B}{f} ,$$

where

$$m = \frac{100c}{B} .$$

We may use the relation (Reference 1):

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$$\left(\frac{S}{N}\right)_{\substack{rms \\ voltage}} = \sqrt{\frac{P_r}{P_N}} = \sqrt{\frac{P_t \ G_t \ G_r \left(\frac{\lambda}{4\pi r}\right)^2}{kT'B'}} \ ,$$

where

P_r = signal power at receiver,

P_N = noise power at receiver,

P_t = power of transmitter,

G_t = power gain of transmitter antenna,

G_r = power gain of receiving antenna,

λ = wavelength of carrier (meters),

r = slant range of satellite (meters),

k = Boltzmann's constant,

T' = sum of disturbing noise temperatures in °K,

B' = post-detection bandwidth.

Then, a general expression relating m with r is

$$m = \frac{100}{B} \left(\frac{T\lambda \sqrt{P_t \ G_t \ G_r}}{2K \sqrt{kT'B'} \ r} - \frac{1}{f} \right)^{-1} . \tag{4}$$

If 1/f is negligible the expression reduces to

$$m = \left(\frac{200K \sqrt{kT'B'}}{B T\lambda \sqrt{P_t \ G_t \ G_r}} \right) r \ . \tag{5}$$

Thus the resolution is directly proportional to the slant range when the term 1/f of Equation 4 is negligible.

Equation 5 shows that m varies as $\sqrt{B'}$ (the post-detection bandwidth, which is directly related to the system noise bandwidth), so that it is advantageous to narrow B', as was mentioned earlier.

Figure 9 illustrates the relation between the resolution in percent (m) and the slant range of the satellite, using the parameters of the S-3 program.* For this particular calculation the measurement interval was chosen at 9 msec, and the error criterion K expressed in units of standard deviation was chosen to equal 3 (i.e., 99.7 percent of the frequency measurements are within the resolution m.) For small values of m the curve terminates at a resolution of 0.0028 percent for 15 kc and 0.00095 percent at 5 kc, because of the quantization error.

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*S-3 is an internal GSFC designation for a series of satellites.

The termination in Figure 9 for large slant ranges (i.e., low S/N) is also limited by the implementation technique. At these large ranges the S/N is low enough to cause more than $2n + 1$ zero-crossing detections per n cycles. Tests indicate that the threshold of these erroneous zero-crossing detections is, with the present equipment, in the neighborhood of S/N = +5 db (range = 65,000 km). For slant ranges beyond this threshold the curve is shown as a dashed line.

CONCLUSIONS

High resolution measurements for a pulse frequency modulation telemetry signal can be made when the S/N is not too low. These measurements are made by detecting the

$$m = \left(\frac{200K \sqrt{KT' B'}}{BT\lambda} \frac{}{\sqrt{P_t G_t G_r}} \right) ,$$

T = 9 msec
K = 3 FOR 99.7% PROBABILITY THAT A DATA POINT HAS THE SPECIFIED RESOLUTION

Figure 9—Resolution of measurement vs. slant range for a typical PFM satellite.

number of zero crossings of the signal and the elapsed time between the first and last crossing in equipment such that the measurement is made over an integral number of cycles, determined by a preset fixed time interval. This technique has been used to obtain significant improvement in measurement resolution when the signal-to-noise ratio permits.

Test results confirm the calculated relations between the resolution, signal-to-noise ratio, signal frequency, and time interval of measurement. Thus, the feasibility of the method chosen has been demonstrated by the experimental equipment, and the validity of theoretical predictions of resolution has been confirmed by realistic simulation. Operational equipment is now being built to exploit this technique.

The resolution of frequency measurements can be related to the slant range of a satellite when the key parameters of that satellite telemetering system are known. Potential users of the high resolution capability should consider this relation when fixing the parameters of the airborne system.

The improvement in accuracy described above could be obtained at lower S/N if the S/N could be improved prior to zero-crossing measurements. A practical method of implementating this is not now available, but development of a system with increased efficiency is progressing.
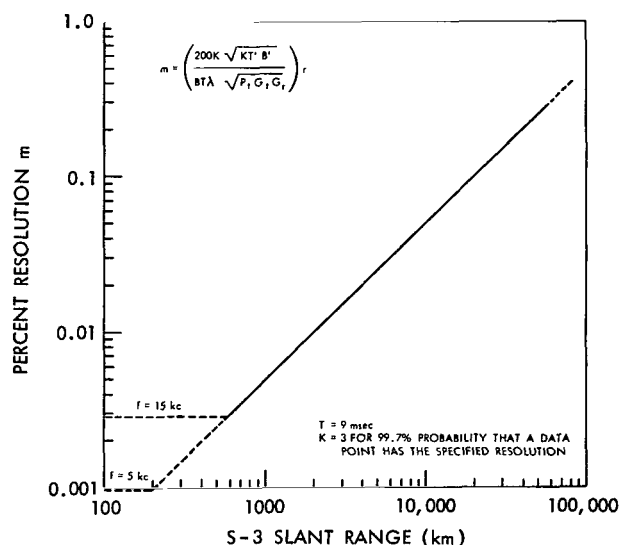
REFERENCES

1. Rochelle, R. W., "Pulse-Frequency-Modulation Telemetry," NASA Technical Report R-189, January 1964.

2. Demmerle, A. M., "On Frequency Measurements and Resolution," NASA Technical Note D-2216, 1964.

3. Data Instrumentation Development Branch, "Operation and Maintenance Manual for Magnetometer Data Processor," Goddard Space Flight Center Document X-544-64-112, May 1964.

Appendix A

Examples of Error Distributions Demonstrating The
Use of The Term "Resolution"

Shown in Figure A1 is a Gaussian probability density function which can be expressed as $p(\gamma) = \left(1/\sigma \sqrt{2\pi}\right) e^{-\gamma^2/2\sigma^2}$ where γ represents the deviation between the frequency as interpreted by the measurement, $\hat{f}$, and the true signal frequency

$$f, \quad \gamma = f - \hat{f}.$$

The probability that the variable γ will lie in the range $-\gamma_1$ to $+\gamma_1$ is

$$\text{prob}\left(-\gamma_1 < \gamma < +\gamma_1\right) = \int_{-\gamma_1}^{+\gamma_1} p(\gamma) \, d\gamma \, ,$$

where

$$\int_{-\infty}^{\infty} p(\gamma) \, d\gamma = 1 \, .$$

Figure A1—Gaussian probability density function.

For the function shown in Figure A1 the shaded area represents the probability that the frequency error is less than $\gamma = \pm\sigma$,

$$\text{prob}(-\sigma < \gamma < +\sigma) = \int_{-\sigma}^{+\sigma} p(\gamma) \, d\gamma = 0.683 \, .$$
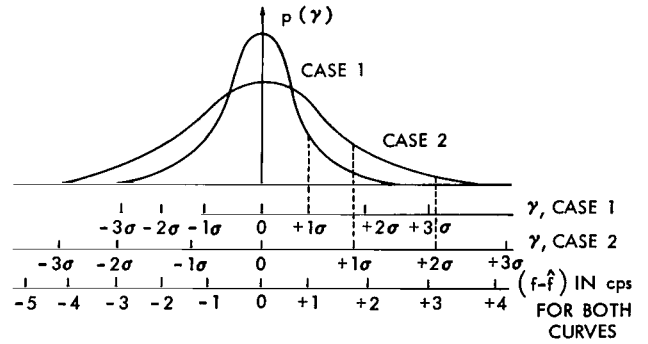
Figure A2—Probability density of error in cps for two S/N values.

Figure A2 illustrates two distributions of errors, where the measurements of $\hat{f}$ were made for different S/N values. Case 1 represents the higher S/N.

For purposes of illustration let $\gamma = \sigma$, of case 1 in Figure A2, represent a frequency reading error of 1 cps. In this case the probability is 0.683 that $f - \hat{f}$ for any one reading, from the
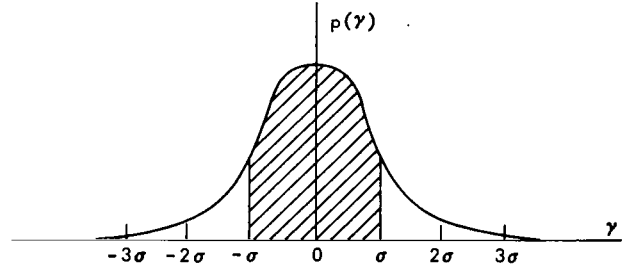
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ensemble of readings, will be within ±1 cps, or that 68.3 percent of all readings will be in error by less than 1 cps. For this same distribution it is also true that the probability is 0.997 that $f - \hat{f}$ is in the range of ±3 cps (i.e., $c = 1$ cps at $K = 1$, or $c = 3$ cps at $K = 3$). For case 2 of Figure A2 the probability, for example, is 0.683 that $f - \hat{f}$ will be within ±1.8 cps or the probability is 0.997 that $f - \hat{f}$ will be within ±4.2 cps (i.e., $c = 1.8$ at $K = 1$, or $c = 4.2$ cps at $K = 3$).

Appendix B

An Analysis of the Quantization Error in
the High Resolution Processor (HRP)

The quantization error for this HRP system can be analyzed with the use of Figure B1. The time interval indicated by z, which is z times 0.2 μsec, is actually too short by an amount u, and too long by an amount v. Because the reference frequency on tape is completely independent of the data frequency, both u and v have a rectangular density distribution as shown in Figure B2.

Figure B1—Determining the counting time by counting the reference cycles.

The total elapsed time measurement ($z \times 0.2$ μsec) is in error by $\Delta = u - v$ which has the probability density distribution shown in Figure B3. From this probability density distribution the value of a particular Δ, say Δ_1, where 99.7 percent of all time readings are less than $|\Delta_1|$, is found to be $\pm 0.19 \times 10^{-6}$ sec.
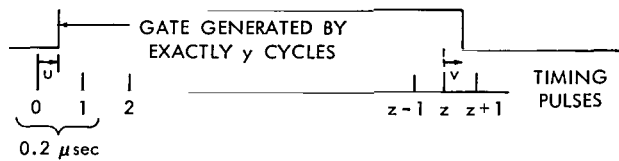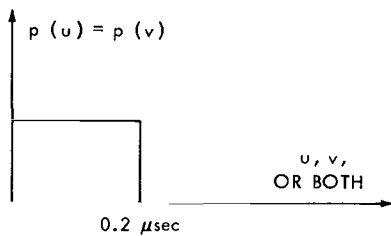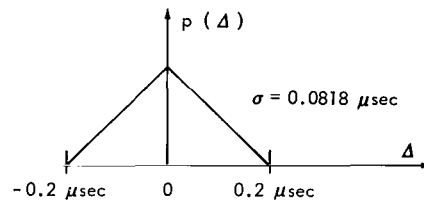

Figure B2—Probability density of u and v.

Figure B3—Probability density of error, u − v.

The frequency is estimated as $\hat{f} = (y/z)(5 \times 10^6)$ cps, so that the error in z reflects itself as a percent error in $\hat{f}$ in accordance with the magnitude of $\hat{f}$. For example, if z is the count accumulated for approximately 10 msec, and z is modified by ϵ where

$$\epsilon = \frac{\Delta_1}{z(0.2 \ \mu\text{sec})} = \frac{0.19 \times 10^{-6} \ \text{sec}}{10 \times 10^{-3} \ \text{sec}} = 0.19 \times 10^{-4} \ ,$$

then the true frequency

$$f = \frac{y \times 5 \times 10^6}{z(1 \mp \epsilon)} = \frac{y}{z} \times 5 \times 10^6 \ (1 \pm \epsilon + \epsilon^2 \pm \epsilon^3 + \epsilon^4 + \cdots) \ ,$$

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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